Armorial Analysis via Semantic Networks:

The Dering and Zurich Rolls

[Preliminary Results]

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The Problem

Researching armory presents interesting and unique challenges. Unlike onomastics research, which is often served by compiling lists of attested names and discovering the comparatively few and generally obvious grammatical patterns among them, armorial research is in a more primitive state: where we try to discern general and predominant patterns across cultures (in the SCA, this is Core Style), as well as patterns less frequently seen in a specific culture (the Individually Attested Patterns), but over data that is at times difficult to represent and frequently "noisy", with deviations from the norms.

Traditional approaches have generally been limited to collecting basic information:

- Frequency of tinctures and tincture combinations
- Frequency of charge types
- Frequency of simple relationships, such as primary groups with tertiary charges

However, getting to these results, even with computer assistance, presents its own challenges:

- *A priori* expectations and bias
- Data representation (including data modelling)
- Data management (including data entry and consistency)
- Queryability

In this paper, I will briefly describe these challenges, and a proposed approach to address them.

A priori expectations

There is a simple truth to be had in all research: Generally speaking, you will only discover what you look for. When it comes to exploration and the search for new things, we need to be careful about how our personal biases and expectations impact our results. The same is true of armorial research.

In the SCA, we are very much biased toward armory that fits into a specific set of relationships. Those relationships are described in *The Standards for Evaluation of Names and Armory, Appendix I: Charge Group Theory* (Society for Creative Anachronism, 2013). They establish a broadly-useful language with which to describe the design of armory, and to discuss how that armory might be altered by cadet branches – the children and their descendants – of a bearer of arms, following practices outlined in Robert Gayre of Gayre and Nigg's work, *Heraldic Cadency: The Development of Differencing of Coats of Arms for Kinsmen and Other Purposes* (Gayre of Gayre and Nigg, 1961).

As a side effect of using our specific language, we become unable to adequately describe, let alone discuss, armory that falls outside of the language's model. This is a large unconscious problem for us, until we attempt to analyze some of the more "pictorial" designs to be found in manuscripts.

I cannot at this time think of a way out of this problem: Whatever language we select will suffer from this same issue, merely changing (potentially reducing) the specific set of problem cases. It is, therefore, something we must simply keep in mind as we research armory.

Data representation

A typical tool for analyzing armory on a computer is a spreadsheet. Spreadsheets have a distinct advantage in being flexible and easy to understand, provided you stick to simple, common uses, such as recording rows of data with various properties and property values. The challenge in this case is in defining the set of properties being recorded.

For example, to cover the cases represented by:

- *Argent.*
- *Argent, a fess sable.*
- *Argent, a fess between three roundels sable.*
- *Argent, on a fess between three roundels sable three mullets argent.*
- *Argent, on a fess between three roundels sable each charged with a fleur-de-lys, three mullets argent.*
- *Argent, a fess between three roundels, all within a bordure sable.*

We soon discover ourselves facing, at a minimum:

1. Field tincture
2. Primary group: Count
3. Primary group: Type
4. Primary group: Tincture
5. Primary group: Tertiary group Count
6. Primary group: Tertiary group Type
7. Primary group: Tertiary group Tincture
8. Secondary group #1: Count
9. Secondary group #1: Type
10. Secondary group #1: Tincture
11. Secondary group #1: Tertiary group Count
12. Secondary group #1: Tertiary group Type
13. Secondary group #1: Tertiary group Tincture
14. Secondary group #2: Count
15. Secondary group #2: Type
16. Secondary group #2: Tincture

Noticing that we have not yet made affordance for partitioned fields, complex lines, multiple types within a charge group, multi-tinctured charges, charge variants, arrangements, postures, or other charge groups, the number of columns of this table is already significant. Just expressing the extent of SCA Core Style leads to even greater numbers, and the ability to read and understand a single row (for example, in validating the data) is greatly impaired.

One specific question to ask when facing a set of columns such as this: What determines which secondary group gets to be #1, and which gets to be #2? What happens when you need to add #3 (e.g., *Argent mullety, a fess between three roundels, all within a bordure sable*)?

This is an example of a rather difficult flaw to overcome in this representation: The implicit and required ranking of features that probably might be considered co-equal.

Data management

Managing data, and writing tools to manage data, is a lucrative facet of the software industry: There is widespread admission that it is simultaneously important to get right, and yet difficult to get right, without overlooking anything. Spreadsheets, grammar and spelling checkers, databases, and search engines all exist because of this basic need. But the data still needs to be related to software, and thus the art of data entry arose. But typos abound, data gets placed in incorrect fields, and inconsistencies arising from simple drift in our mental models as we enter data. Proofreading is important, and we can write self-validating forms to help address the more common issues, but to build upon a case above: What happens when we discover a third secondary charge group, forcing us to rewrite a form? What happens when partway through encoding a dataset we shift, unconsciously, how we order secondary charge groups – a situation a computer will not notice?

Obviously, what *should* happen is revisiting all the prior data and ensuring it is updated to reflect the revised model. In a small data set this is trivial, but in any moderately sized data set, we easily reach hundreds of cases. In a large data set they number in the thousands.

Humans are inconsistent in their labor, but computers are very consistent, if they are told what to do: Data management, and the consistent processing of data, is their reason for existence. Where possible, computers should be tackling the more complex and laborious aspects of data entry and processing: humans should be tasked with human work, describing to computers the rules they want to apply in processing data.

Queryability

Assuming data is successfully (and after much effort) captured in a spreadsheet, what can you do with it? You could do some simple histograms of specific columns to get, say, the frequency of field tinctures. Those more practiced with common spreadsheets can create a pivot chart of pairs of tinctures in partitioned armory. You might have difficulty going beyond two, but it could be done in theory.

That said, how easily could you answer the question:

> *If a tertiary mullet is argent, how often is there a fess between two chevrons?*

Here we discover another problem in modelling data: Not only does our language and model limit the way we can express data, but it limits the questions we may ask of it. A table could be designed that could contain all the information necessary to answer the question posed above, but only if we know ahead of time that we may want to ask that question.

If we are instead attempting to discover things we have not seen before, we need a data model which yields to asking complex questions.

The Technique

Given the challenges facing traditional tools and the approaches around them, I set out to write software to address those challenges. Specifically, I sought a new approach which focused on several specific factors:

- Increasing the manageability and flexibility of the data model
- Reducing human labor through automation
- Reducing need for establishing target questions *a priori*
- Increasing the range of queries that may be asked of the data

The software was written in the C# programming language, using a SQLite in-process database. The development hardware was a Microsoft Surface Laptop 2 with an Intel Core i7-8650U CPU and 16 GB of RAM.

Data model

A primary complaint about tabular data, such as those found in spreadsheets, regards their management. As described earlier, attempts to describe complex models translate into a dramatic proliferation in the number of columns, and the effort needed to maintain them and use them consistently. Further, as the number of columns increases, the ease of querying the data also increases. However, there is a way to model data that can express complex, arbitrary relationships while also being amenable to complex queries about those relationships: that model is called a *semantic network* (Wikipedia).

Semantic networks describe *relationships* between various *concepts*. A family tree is one common type of semantic network, where the concepts happen to be people, and the relationships are parent-child. There are many semantic networks out there, each one describing sets of facts about some domain in the world.

There is a standard notation to represent semantic networks, based on *triples* – facts in three parts – which includes information about the starting concept (*subject*), the ending concept (*object*), and the type of relationship it represents (*predicate*). In this document, we will write individual triples as:

$$subject \xrightarrow{predicate} object$$

Subject and *object* each refer to individual concepts, represented as $\langle n \rangle$, where *n* may be an abstract identifier or a concrete label. Additionally, *object* may instead refer to a value, such as a name, or an idea, or a count.

This notation allows us to represent knowledge as a set of triples. For example, the statement, *Bartolo Sassoferrato was a lawyer who wrote a treatise on heraldry*, may be represented as the set of facts:

$$\langle 0 \rangle \xrightarrow{has\ name} Bartolo\ de\ Sassoferrato$$

$$\langle 0 \rangle \xrightarrow{has\ profession} lawyer$$

$$\langle 0 \rangle \xrightarrow{wrote} \langle 1 \rangle$$

$$\langle 1 \rangle \xrightarrow{is\ a} treatise$$

$$\langle 1 \rangle \xrightarrow{has\ topic} heraldry$$

This may be read as the following set of statements:

1. Thing $\langle 0 \rangle$ HAS NAME $Bartolo\ de\ Sassoferrato$.
2. Thing $\langle 0 \rangle$ HAS PROFESSION $lawyer$.
3. Thing $\langle 0 \rangle$ WROTE Thing $\langle 1 \rangle$.
4. Thing $\langle 1 \rangle$ IS A $treatise$.
5. Thing $\langle 1 \rangle$ HAS TOPIC $heraldry$.

Given this representation, we can answer complex questions by searching for interconnected groups of facts:

Query	Plan	Answer
What are the names of lawyers?	Find all X and Y such that: $\langle X \rangle \xrightarrow{has\ profession} lawyer$ $\langle X \rangle \xrightarrow{has\ name} Y$	Solutions: $X = \langle 0 \rangle$ $Y = Bartolo\ de\ Sassoferrato$ $Answer\ =\ Bartolo\ de\ Sassoferrato$
What were the professions of those who wrote musical scores about heraldry?	Find all X and Y such that: $\langle X \rangle \xrightarrow{has\ profession} Y$ $\langle X \rangle \xrightarrow{wrote} \langle Z \rangle$ $\langle Z \rangle \xrightarrow{is\ a} musical\ score$ $\langle Z \rangle \xrightarrow{has\ topic} heraldry$	Solutions: *none* (there are no musical scores at all, let alone musical scores satisfying the given constraints) $Answer\ =\ none$

For armory, we shall use the following general schema based on SCA Charge Group Theory:

$$\langle Armory \rangle \xrightarrow{has\ field} \boxed{field}$$

$$\langle Armory \rangle \xrightarrow{has\ primary\ group} \boxed{group}$$

$$\langle Armory \rangle \xrightarrow{has\ secondary\ group} \boxed{group}$$

$$\langle Armory \rangle \xrightarrow{has\ debruising\ group} \boxed{group}$$

$$\boxed{group} \xrightarrow{has\ tertiary\ group} \boxed{group'}$$

$$\boxed{field} \xrightarrow{has\ partition\ type} type$$

$$\boxed{field} \xrightarrow{has\ partition\ treatment} treatment$$

$$\boxed{field} \xrightarrow{has\ partition} \boxed{partition}$$

$$\boxed{partition} \xrightarrow{has\ tincture} tincture$$

$$\boxed{partition} \xrightarrow{has\ tincture\ class} class$$

$$\boxed{group} \xrightarrow{includes\ charges} \boxed{charges}$$

$$\boxed{charges} \xrightarrow{has\ count} count$$

$$\boxed{charges} \xrightarrow{has\ arrangement} count$$

$$\boxed{charges} \xrightarrow{has\ charge\ type} type$$

$$\boxed{charges} \xrightarrow{has\ charge\ class} class$$

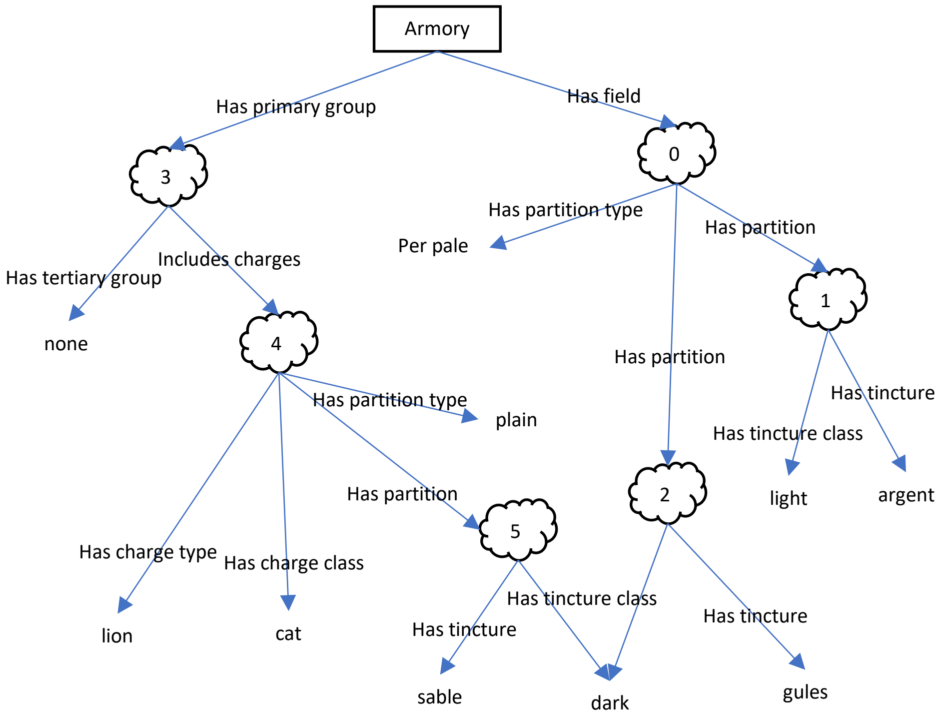
$$\boxed{charges} \xrightarrow{has\ treatment} treatment$$

$$\boxed{charges} \xrightarrow{has\ posture} posture$$

$$\boxed{charges} \xrightarrow{has\ partition\ type} type$$

$$\boxed{charges} \xrightarrow{has\ partition} partition$$

For example, we may describe *Per pale argent and gules, three lions sable* in part as the network:



And as the following set of triples (adding a few more facts, such as the lack of secondary or debruising groups, and the count, arrangement, and posture of the primary charges):

$$\langle Armory\rangle \xrightarrow{has\ field} \langle 0\rangle$$
$$\langle Armory\rangle \xrightarrow{has\ primary\ group} \langle 3\rangle$$
$$\langle Armory\rangle \xrightarrow{has\ secondary\ group} none$$
$$\langle Armory\rangle \xrightarrow{has\ debruising\ group} none$$
$$\langle 0\rangle \xrightarrow{has\ partition\ type} per\ pale$$
$$\langle 0\rangle \xrightarrow{has\ partition} \langle 1\rangle$$
$$\langle 1\rangle \xrightarrow{has\ tincture} argent$$
$$\langle 1\rangle \xrightarrow{has\ tincture\ class} light$$
$$\langle 0\rangle \xrightarrow{has\ partition} \langle 2\rangle$$
$$\langle 2\rangle \xrightarrow{has\ tincture} gules$$
$$\langle 2\rangle \xrightarrow{has\ tincture\ class} dark$$
$$\langle 3\rangle \xrightarrow{has\ tertiary\ group} none$$
$$\langle 3\rangle \xrightarrow{includes\ charges} \langle 4\rangle$$
$$\langle 4\rangle \xrightarrow{has\ count} three$$
$$\langle 4\rangle \xrightarrow{has\ arrangement} two\ and\ one$$
$$\langle 4\rangle \xrightarrow{has\ charge\ type} lion$$
$$\langle 4\rangle \xrightarrow{has\ posture} rampant$$
$$\langle 4\rangle \xrightarrow{has\ charge\ class} cat$$
$$\langle 4\rangle \xrightarrow{has\ partition\ type} plain$$
$$\langle 4\rangle \xrightarrow{has\ partition} \langle 5\rangle$$
$$\langle 5\rangle \xrightarrow{has\ tincture} sable$$
$$\langle 5\rangle \xrightarrow{has\ tincture\ class} dark$$

Translating blazon to semantic network

As input to the analysis software, I chose to start with blazon. Blazon is easy to produce and useful in discussing armory, both as a referent to the armory itself as well as a description of the armory's design. That, then, begs the question of how we translate it, once we have it.

For the project, I use a brute force method of interpreting blazon. The first step is *normalization*, taking specific words or phrases and rewriting them into a more common, standardized form. Plurals, for example, are rewritten to their singular form, and certain shortcut phrases rewritten (e.g., *mullety* becomes *seme of mullet)*. This is done to reduce the number of cases needed to describe the entire set of armory.

Then, given a lexicon of words and phrases, and nominal "part of speech" tokens to associate with them, a *tokenizer* sequentially and starting from the beginning of the blazon, each word or phrase is marked with the associated part of speech. If at any point no appropriate matches are found, an error is raised to improve the lexicon.

For example, the blazon *Argent mullety, three lions passant gules*, becomes the sequence:

1. `TINCTURE` (Argent)
2. `COUNT_SEME` (seme)
3. `PREPOSITION` (of)
4. `TYPE` (mullet)
5. `COUNT` (three)
6. `TYPE` (lion)
7. `POSTURE` (passant)
8. `TINCTURE` (gules)

The parts of speech are then catenated together to form a fingerprint of the blazon:

```
TINCTURE COUNT_SEME PREPOSITION TYPE COUNT TYPE POSTURE TINCTURE
```

In the next step, an additional list is used to map these fingerprints to templates used to generate the appropriate network. We rely on the formulaic way we produce blazon, and the idea that there has historically been a coherent way in which armory is designed (the fact of which allows the SCA to define a Core Style) to keep the number of different cases significantly less than the number of blazons we need to cover.

```
TINCTURE COUNT_SEME PREPOSITION TYPE COUNT TYPE POSTURE TINCTURE
```

$\langle Armory\rangle \xrightarrow{has\ field} \langle 0\rangle$

$\langle Armory\rangle \xrightarrow{has\ primary} \langle 3\rangle$

$\langle Armory\rangle \xrightarrow{has\ secondary} \langle 2\rangle$

$\langle Armory\rangle \xrightarrow{has\ debruising} none$

$\langle 0\rangle \xrightarrow{has\ partition\ type} plain$

$\langle 0\rangle \xrightarrow{has\ partition} \langle 1\rangle$

$\langle 1\rangle \xrightarrow{has\ tincture} argent$

$\langle 1\rangle \xrightarrow{has\ tincture\ class} light$

$\langle 2\rangle \xrightarrow{has\ count} seme$

$\langle 2\rangle \xrightarrow{has\ arrangement} strewn$

$\langle 2\rangle \xrightarrow{has\ charge\ type} star$

$\langle 2\rangle \xrightarrow{has\ charge\ class} star$

$\langle 2\rangle \xrightarrow{has\ partition\ type} plain$

$\langle 2\rangle \xrightarrow{has\ partition} \langle 6\rangle$

$$\langle 6\rangle \xrightarrow{\textit{has tincture}} \textit{gules}$$

$$\langle 3\rangle \xrightarrow{\textit{has tertiary}} \textit{none}$$

$$\langle 3\rangle \xrightarrow{\textit{includes charges}} \langle 4\rangle$$

$$\langle 4\rangle \xrightarrow{\textit{has count}} \textit{three}$$

$$\langle 4\rangle \xrightarrow{\textit{has arrangement}} \textit{two and one}$$

$$\langle 4\rangle \xrightarrow{\textit{has charge type}} \textit{lion}$$

$$\langle 4\rangle \xrightarrow{\textit{has posture}} \textit{passant}$$

$$\langle 4\rangle \xrightarrow{\textit{has charge class}} \textit{cat}$$

$$\langle 4\rangle \xrightarrow{\textit{has partition type}} \textit{plain}$$

$$\langle 4\rangle \xrightarrow{\textit{has partition}} \langle 5\rangle$$

$$\langle 5\rangle \xrightarrow{\textit{has tincture}} \textit{gules}$$

$$\langle 5\rangle \xrightarrow{\textit{has tincture class}} \textit{dark}$$

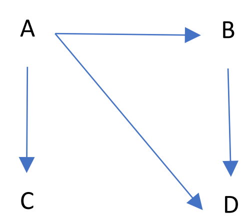
Enumerating all patterns

If we were to take the network above and put it into any of the available graph database engines (a semantic network being an example of a *graph*), we'd be able to ask the database to return all the items matching specific patterns, such as we were able to do with Bartolo de Sassoferrato and his treatise. With some additional rules added to the database itself, we could infer certain traits like "field primary" or determine a value for "complexity count". We could, for example, represent the SCA Ordinary and Armorial in that manner, and greatly change how conflict checks were performed, returning all and only those items that matched.

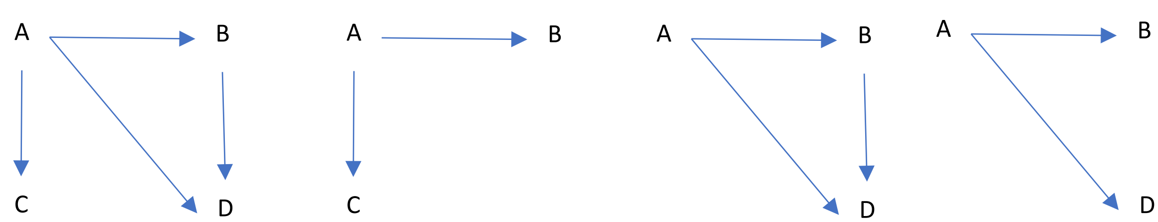
However, that would not serve the goal of reducing the need for *a priori* information. While searching semantic and graph data for common patterns ("motifs") is itself an active area of computer science, the specific desire here is to in fact find all patterns, no matter how uncommon. That leads to another brute force approach: systematically *enumerating* every pattern within each individual network.

This is a very computationally intensive and time-consuming task.

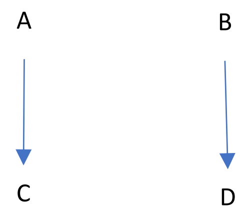
Omitting implementation-specific details, it is sufficient to describe the effect as finding every set of relationships where, if you reconnect each of them, you end up with a single network. For example, given this network:



Then these are considered valid patterns because they are connected into a single whole:



While this disconnected set of relationships is not considered a valid pattern:



As mentioned, this is a computationally intensive undertaking, and the number of found patterns greatly increases with the complexity of the item under consideration. For modest numbers of edges, the number of unique patterns to be found in it can be vast: In the present study, 29,472,719 patterns resulted from considering 807 items, each one with up to two dozen or so edges.

Before storing in a database, each pattern undergoes a *canonicalization* process, which rewrites the set of triples in such a way that, if two different networks have the same set of concepts, and the same set of relationships between them (that is, if they are *isomorphic*), then the set of triples will have the same order. Doing so permits the database to find matching patterns scattered throughout the entire dataset. The process itself is outside the scope of this document, but is based on that outlined in (Hogan, 2015).

The Result

With all canonicalized patterns associated with their source armory now stored in the database, it becomes straightforward to then generate a report of all sets of armory that contain any given pattern:

```
CREATE TABLE ArmoryIdsByPattern
AS
SELECT
    Pattern,
    Min(PatternId) AS PatternId,
    Count(ArmoryId) AS ArmoryCount,
    GROUP_CONCAT(ArmoryId, ' ') as ArmoryIds
FROM (
    SELECT Pattern, ArmoryId, PatternId
    FROM Patterns
    ORDER BY Pattern, ArmoryId
    )
GROUP BY Pattern
ORDER BY ArmoryIds;
```

Naively, such a query would produce thousands of rows. The data returned would be correct and useful for our purposes. However, if we continue with that data and produce a report of what *sets* of armory patterns are shared by what *sets* of armory, we can cull redundancies within a set of patterns, by removing those contained in other patterns already present. In the present study, this reduces the result from over 29 million rows in the output, to 14,309.

Further, if each piece of armory is associated with metadata describing its origins, whether regionally, such as "England" versus "Germany", or temporally – "13th century" vs "14th century" – we can pre-compute the fraction of items present in each row of the results and discover patterns that are significantly more common in one culture versus another. For example:

```
Frequency difference: 33.7%

Pattern: {
(<0>, tertiary group, false)
(<0>, has charge class, Ordinary)
(<0>, has partition type, Plain)
(<1>, includes charges, <0>)
}

Pattern frequency: 287/807 (36%)

..[region:England] => 181/325 (56%)
..[region:Germany] => 106/482 (22%)
..[time:late-13c] => 181/325 (56%)
..[time:mid-14c] => 106/482 (22%)
..[region:England; time:late-13c] => 181/325 (56%)
..[region:Germany; time:mid-14c] => 106/482 (22%)
```

This tells us that the pattern, *a charge group of any sort which contains any number of uncharged, plain-tinctured ordinaries*, can be found in 56% of the items from the English source, but in only 22% of the German items. Appendix B contains a list of such patterns where the frequency difference is 20% or greater.

Results of the Dering Roll vs Zurich Roll comparison

The appendices at the end of this paper show the primary results of applying this analysis to the Dering and Zurich Rolls, two rolls created in different regions of Europe, within roughly a century of each other. From the Dering Roll, I used the blazons of 325 items previously identified by Cormac Mór (Rhodes, 2015). Of the Zurich Roll, 482 blazons collected by Gunnvör silfrahárr were used (Ward, n.d.).

Appendix A: Frequencies for specific tinctures

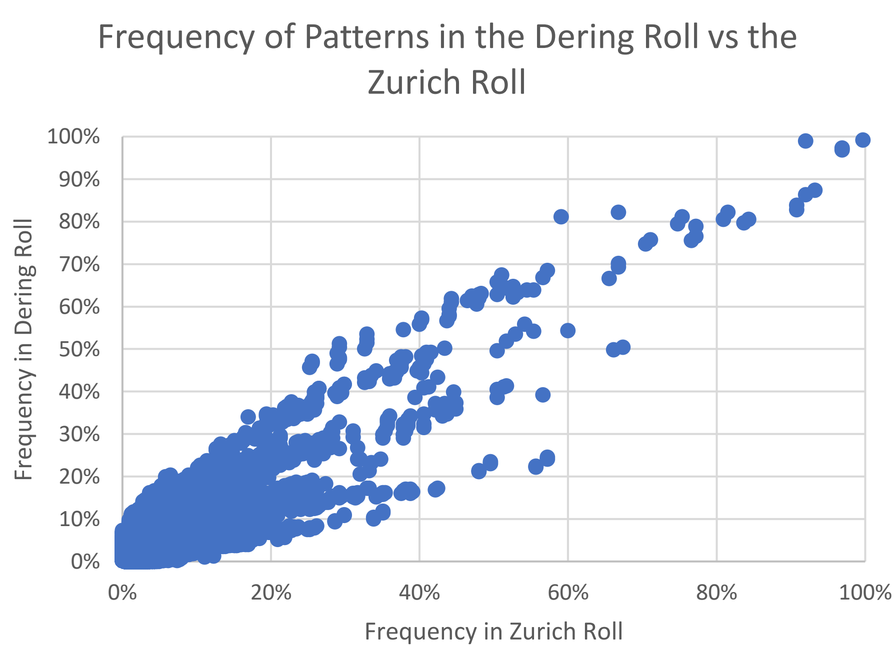
This presents the relative frequencies of tinctures and tincture classes (i.e., light, dark, neutral, fur). It shows that, while it is true that tinctures that are rare in one culture and generally rare in the other, there are still biases to be seen. For example, gules is found in 67% of the Dering Roll items, but only in 50% of those in the Zurich Roll. Similarly, ermine is nearly unknown (N=2) in the Zurich Roll, while found in 8% of the Dering Roll.

Appendix B: Patterns where frequency difference > 20%

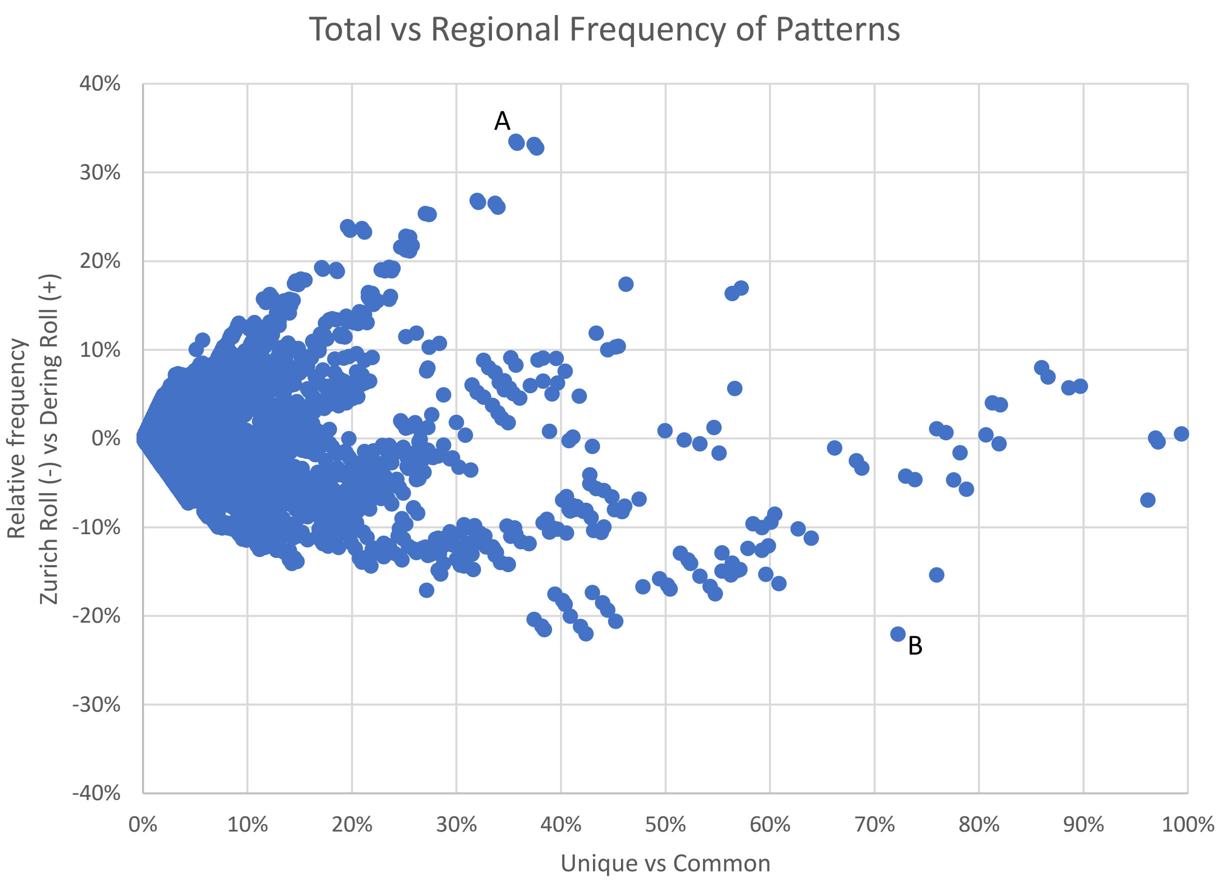
The purpose of this query was to find ways in which the two cultures differed in their design choices. Twenty percent was selected as an arbitrary cutoff after looking at the full results.

The single pattern showing the greatest difference in frequency is *a charge group containing one or more uncharged, plain-tinctured ordinaries* (and potentially other types of charges), occurring in 56% of the Dering Roll items, and only 22% of the Zurich Roll. Other, similar patterns, such as allowing the ordinaries to be charged, appear at similar rates.

Across all patterns, plotting the individual frequencies from each roll shows, overall, a fair amount of agreement between the two cultures. That is, there is a shared idea of armorial design. However, the agreement is not perfect, with some patterns found more often in one culture than in the other:



To further highlight this idea, we can plot the frequency difference between the rolls, versus the overall frequency in the data set. Doing so can give us an idea of the bias between the two cultures, versus how common the pattern is overall.



Here we see that most of the patterns that are found in roughly the same rates between cultures are also found in roughly 20-30% of the overall dataset. Small, simple patterns with little otherwise to distinguish themselves occupy that region of the plot. These include such patterns as *X has tincture argent*. Items that are the most different between the two are found in most of the items overall, so we can feel confident that the bias is not due to simple noise.

For reference, the topmost cluster *A* corresponds to the pattern described earlier as *a charge group containing one or more uncharged, plain-tinctured ordinaries*. Interestingly, the related pattern that is formed by further asserting *no debruising charge group* is found as the bottommost point *B*, both substantially more common across the entire dataset and the Zurich Roll in particular. The other points lying outside of the ±20% frequency difference band are found in Appendix B.

Challenges and Future Work

While personally interesting, this approach is of little impact unless it can be refined and packaged for others to use. To that end, a few things need to be addressed:

- Defining the part of speech map
  - The original software defined all the lexicon and parts of speech in source. It was desirable to move that to a separate, external text file to be passed into the analysis engine. This work has been completed.
- Defining the map from tokenized blazon patterns to semantic networks
  - This work is under development. The current mappings are hardcoded into the software, requiring the user to modify the source code to make changes. It is desirable to move this into a separate text file that becomes one of the inputs into the process.
- Memory requirements
  - At peak, the analysis consumes upwards to 15GB of RAM. Further work is needed to determine how to reduce the amount of RAM needed to a lower, more scalable amount.
  - Similarly, the final size of the SQLite database rises to approximately 50GB. Finding a more compact method of storing the data would be useful.
- Time requirements
  - The analysis of the Dering and Zurich rolls consistently takes around 36 hours on a Microsoft Surface Laptop 2. Further work is needed to determine how to increase the time efficiency of the process. Current efforts are focused on porting the analysis software from C# to the Rust programming language, to take advantage of a potential 6-8x factor in speed without increasing the risk of instability often seen in native implementations. Additionally, investigation into how to increase the efficiency of the final processing steps – finding the sets of patterns common to the sets of armory and culling unnecessary patterns from those sets – is ongoing.

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Appendix A: Frequencies for specific tinctures

The following table illustrates the relative frequency of tinctures in the Dering and Zurich rolls, individually and combined.

Pattern	**Total** **N = 807**	**Dering** **N = 325**	**Zurich** **N = 482**
$\langle 0 \rangle \xrightarrow{has\ tincture\ class} Dark$	802 (99%)	324 (100%)	478 (99%)
$\langle 0 \rangle \xrightarrow{has\ tincture\ class} Light$	784 (97%)	315 (97%)	469 (97%)
$\langle 0 \rangle \xrightarrow{has\ tincture\ class} Neutral$	19 (2%)	15 (5%)	4 (1%)
$\langle 0 \rangle \xrightarrow{has\ tincture\ class} Fur$	46 (6%)	40 (12%)	6 (1%)
$\langle 0 \rangle \xrightarrow{has\ tincture} Argent$	483 (60%)	171 (53%)	312 (65%)
$\langle 0 \rangle \xrightarrow{has\ tincture} Or$	365 (45%)	167 (51%)	198 (41%)
$\langle 0 \rangle \xrightarrow{has\ tincture} Sable$	232 (29%)	66 (20%)	166 (34%)
$\langle 0 \rangle \xrightarrow{has\ tincture} Gules$	462 (57%)	219 (67%)	243 (50%)
$\langle 0 \rangle \xrightarrow{has\ tincture} Azure$	220 (27%)	104 (32%)	116 (24%)
$\langle 0 \rangle \xrightarrow{has\ tincture} Vert$	45 (6%)	4 (1%)	41 (9%)
$\langle 0 \rangle \xrightarrow{has\ tincture} Purpure$	1 (0%)	1 (0%)	0 (0%)
$\langle 0 \rangle \xrightarrow{has\ tincture} Ermine$	27 (3%)	25 (8%)	2 (0%)
$\langle 0 \rangle \xrightarrow{has\ tincture} Vair$	19 (2%)	15 (5%)	4 (1%)

Appendix B: Patterns where frequency difference > 20%

The following table illustrates the patterns for which the Dering and Zurich rolls show an absolute difference in frequency of at least 20%:

$$\Delta f \ = |Dering - Zurich|$$

$$\Delta f \ \geq 20\%$$

Pattern	Total N = 807	Dering N = 325	Zurich N = 482	Δf
$\langle 0 \rangle \xrightarrow{tertiary\ group} \emptyset$ $\langle 0 \rangle \xrightarrow{has\ charge\ class} Ordinary$ $\langle 0 \rangle \xrightarrow{has\ partition\ type} Plain$ $\langle 1 \rangle \xrightarrow{includes\ charges} \langle 0 \rangle$ *Charge groups with one or more uncharged, plain-tinctured ordinaries*	287 (36%)	181 (56%)	106 (22%)	33.7%
$\langle 0 \rangle \xrightarrow{has\ charge\ class} Ordinary$ $\langle 0 \rangle \xrightarrow{has\ partition\ type} Plain$ $\langle 1 \rangle \xrightarrow{includes\ charges} \langle 0 \rangle$ *Charge groups with one or more plain-tinctured ordinaries*	288 (36%)	181 (56%)	107 (22%)	33.5%
$\langle 0 \rangle \xrightarrow{tertiary\ group} \emptyset$ $\langle 0 \rangle \xrightarrow{has\ charge\ class} Ordinary$ $\langle 1 \rangle \xrightarrow{includes\ charges} \langle 0 \rangle$ *Charge groups with one or more uncharged ordinaries*	301 (37%)	186 (57%)	115 (24%)	33.4%
$\langle 0 \rangle \xrightarrow{has\ charge\ class} Ordinary$ $\langle 1 \rangle \xrightarrow{includes\ charges} \langle 0 \rangle$ *Charge groups with one or more ordinaries*	303 (38%)	186 (57%)	117 (24%)	33.0%

Pattern	Total N = 807	Dering N = 325	Zurich N = 482	Δf
$\langle 0 \rangle \xrightarrow{has\ count} 1$ $\langle 0 \rangle \xrightarrow{tertiary\ group} \emptyset$ $\langle 0 \rangle \xrightarrow{has\ charge\ class} Ordinary$ $\langle 0 \rangle \xrightarrow{has\ partition\ type} Plain$ $\langle 1 \rangle \xrightarrow{includes\ charges} \langle 0 \rangle$ *Charge groups with single, uncharged, plain-tinctured ordinaries*	257 (32%)	156 (48%)	101 (21%)	27.0%
$\langle 0 \rangle \xrightarrow{has\ count} 1$ $\langle 0 \rangle \xrightarrow{has\ charge\ class} Ordinary$ $\langle 0 \rangle \xrightarrow{has\ partition\ type} Plain$ $\langle 1 \rangle \xrightarrow{includes\ charges} \langle 0 \rangle$ *Charge groups with single, plain-tinctured ordinaries*	258 (32%)	156 (48%)	102 (21%)	26.8%
$\langle 0 \rangle \xrightarrow{has\ count} 1$ $\langle 0 \rangle \xrightarrow{tertiary\ group} \emptyset$ $\langle 0 \rangle \xrightarrow{has\ charge\ class} Ordinary$ $\langle 1 \rangle \xrightarrow{includes\ charges} \langle 0 \rangle$ *Charge groups with single, uncharged ordinaries*	271 (34%)	161 (50%)	110 (23%)	26.7%
$\langle 0 \rangle \xrightarrow{has\ count} 1$ $\langle 0 \rangle \xrightarrow{has\ charge\ class} Ordinary$ $\langle 1 \rangle \xrightarrow{includes\ charges} \langle 0 \rangle$ *Charge groups with single ordinaries*	273 (34%)	161 (50%)	112 (23%)	26.3%

Pattern	Total N = 807	Dering N = 325	Zurich N = 482	Δf
$\langle 0\rangle \xrightarrow{tertiary\ group} \emptyset$ $\langle 0\rangle \xrightarrow{has\ charge\ class} Ordinary$ $\langle 0\rangle \xrightarrow{has\ partition\ type} Plain$ $\langle 1\rangle \xrightarrow{includes\ charges} \langle 0\rangle$ $\langle Armory\rangle \xrightarrow{primary\ group} \langle 1\rangle$ *Armory with a primary group that includes one or more uncharged, plain-tinctured ordinaries*	218 (27%)	138 (42%)	80 (17%)	25.9%
$\langle 0\rangle \xrightarrow{tertiary\ group} \emptyset$ $\langle 0\rangle \xrightarrow{has\ charge\ class} Ordinary$ $\langle 1\rangle \xrightarrow{includes\ charges} \langle 0\rangle$ $\langle Armory\rangle \xrightarrow{primary\ group} \langle 1\rangle$ *Armory with a primary group that includes one or more uncharged ordinaries*	221 (27%)	139 (43%)	82 (17%)	25.8%
$\langle 0\rangle \xrightarrow{has\ tincture\ class} Dark$ $\langle 1\rangle \xrightarrow{has\ partition} \langle 0\rangle$ $\langle 1\rangle \xrightarrow{tertiary\ group} \emptyset$ $\langle 1\rangle \xrightarrow{has\ charge\ class} Ordinary$ $\langle 1\rangle \xrightarrow{has\ partition\ type} Plain$ $\langle 2\rangle \xrightarrow{includes\ charges} \langle 1\rangle$ *Charge groups with one or more uncharged, dark, plain-tinctured ordinaries*	157 (19%)	110 (34%)	47 (10%)	24.1%
$\langle 0\rangle \xrightarrow{has\ tincture\ class} Dark$ $\langle 1\rangle \xrightarrow{has\ partition} \langle 0\rangle$ $\langle 1\rangle \xrightarrow{tertiary\ group} \emptyset$ $\langle 1\rangle \xrightarrow{has\ charge\ class} Ordinary$ $\langle 2\rangle \xrightarrow{includes\ charges} \langle 1\rangle$ *Charge groups with one or more uncharged ordinaries that have a dark partition or are dark, plain-tinctured*	168 (21%)	114 (35%)	54 (11%)	23.9%

Pattern	Total N = 807	Dering N = 325	Zurich N = 482	Δf
$\langle 0\rangle \xrightarrow{has\ tincture\ class} Dark$ $\langle 1\rangle \xrightarrow{has\ partition} \langle 0\rangle$ $\langle 1\rangle \xrightarrow{has\ charge\ class} Ordinary$ $\langle 1\rangle \xrightarrow{has\ partition\ type} Plain$ $\langle 2\rangle \xrightarrow{includes\ charges} \langle 1\rangle$ *Charge groups with one or more dark, plain-tinctured ordinaries*	159 (20%)	110 (34%)	49 (10%)	23.7%
$\langle 0\rangle \xrightarrow{has\ tincture\ class} Dark$ $\langle 1\rangle \xrightarrow{has\ partition} \langle 0\rangle$ $\langle 1\rangle \xrightarrow{has\ charge\ class} Ordinary$ $\langle 2\rangle \xrightarrow{includes\ charges} \langle 1\rangle$ *Charge groups with one or more ordinaries that have a dark partition or are dark, plain-tinctured*	170 (21%)	114 (35%)	56 (12%)	23.5%
$\langle 0\rangle \xrightarrow{tertiary\ group} \emptyset$ $\langle 0\rangle \xrightarrow{has\ charge\ class} Ordinary$ $\langle 0\rangle \xrightarrow{has\ partition\ type} Plain$ $\langle 1\rangle \xrightarrow{has\ partition\ type} Plain$ $\langle 2\rangle \xrightarrow{includes\ charges} \langle 0\rangle$ $\langle Armory\rangle \xrightarrow{has\ field} \langle 1\rangle$ $\langle Armory\rangle \xrightarrow{primary\ group} \langle 2\rangle$ *Armory with a plain field and a primary group of one or more uncharged, plain-tinctured ordinaries*	202 (25%)	126 (39%)	76 (16%)	23.0%
$\langle 0\rangle \xrightarrow{tertiary\ group} \emptyset$ $\langle 0\rangle \xrightarrow{has\ charge\ class} Ordinary$ $\langle 1\rangle \xrightarrow{has\ partition\ type} Plain$ $\langle 2\rangle \xrightarrow{includes\ charges} \langle 0\rangle$ $\langle Armory\rangle \xrightarrow{has\ field} \langle 1\rangle$ $\langle Armory\rangle \xrightarrow{primary\ group} \langle 2\rangle$ *Armory with a plain field and a primary group of one or more uncharged ordinaries*	205 (25%)	127 (39%)	78 (16%)	22.9%

Pattern	Total N = 807	Dering N = 325	Zurich N = 482	Δf
$\langle 0\rangle \xrightarrow{tertiary\ group} \emptyset$ $\langle 0\rangle \xrightarrow{has\ charge\ class} Ordinary$ $\langle 0\rangle \xrightarrow{has\ partition\ type} Plain$ $\langle 1\rangle \xrightarrow{includes\ charges} \langle 0\rangle$ $\langle Armory\rangle \xrightarrow{primary\ group} \langle 1\rangle$ $\langle Armory\rangle \xrightarrow{debruising\ group} \emptyset$ *Armory with a primary group of one or more uncharged, plain-tinctured ordinaries and no debruising group*	205 (25%)	126 (39%)	79 (16%)	22.4%
$\langle 0\rangle \xrightarrow{tertiary\ group} \emptyset$ $\langle 0\rangle \xrightarrow{has\ charge\ class} Ordinary$ $\langle 1\rangle \xrightarrow{includes\ charges} \langle 0\rangle$ $\langle Armory\rangle \xrightarrow{primary\ group} \langle 1\rangle$ $\langle Armory\rangle \xrightarrow{debruising\ group} \emptyset$ *Armory with a primary group of one or more uncharged ordinaries and no debruising group*	208 (26%)	127 (39%)	81 (17%)	22.3%
$\langle 0\rangle \xrightarrow{has\ charge\ class} Central\ ordinary$ $\langle 0\rangle \xrightarrow{tertiary\ group} \emptyset$ $\langle 0\rangle \xrightarrow{has\ charge\ class} Ordinary$ $\langle 0\rangle \xrightarrow{has\ partition\ type} Plain$ $\langle 1\rangle \xrightarrow{includes\ charges} \langle 0\rangle$ $\langle Armory\rangle \xrightarrow{primary\ group} \langle 1\rangle$ *Armory with a primary group of one or more uncharged, plain-tinctured central ordinaries*	199 (25%)	122 (38%)	77 (16%)	21.6%
$\langle 0\rangle \xrightarrow{has\ charge\ class} Central\ ordinary$ $\langle 0\rangle \xrightarrow{tertiary\ group} \emptyset$ $\langle 0\rangle \xrightarrow{has\ charge\ class} Ordinary$ $\langle 1\rangle \xrightarrow{includes\ charges} \langle 0\rangle$ $\langle Armory\rangle \xrightarrow{primary\ group} \langle 1\rangle$ *Armory with a primary group of one or more uncharged central ordinaries*	202 (25%)	123 (38%)	79 (16%)	21.5%

Pattern	Total N = 807	Dering N = 325	Zurich N = 482	Δf
$\langle 0\rangle \xrightarrow{has\ charge\ class} Central\ ordinary$ $\langle 0\rangle \xrightarrow{tertiary\ group} \emptyset$ $\langle 0\rangle \xrightarrow{has\ charge\ class} Ordinary$ $\langle 0\rangle \xrightarrow{has\ partition\ type} Plain$ $\langle 1\rangle \xrightarrow{includes\ charges} \langle 0\rangle$ *Charge groups with one or more uncharged, plain-tinctured central ordinaries*	203 (25%)	123 (38%)	80 (17%)	21.2%
$\langle 0\rangle \xrightarrow{has\ charge\ class} Central\ ordinary$ $\langle 0\rangle \xrightarrow{tertiary\ group} \emptyset$ $\langle 0\rangle \xrightarrow{has\ charge\ class} Ordinary$ $\langle 1\rangle \xrightarrow{includes\ charges} \langle 0\rangle$ *Charge groups with one or more uncharged central ordinaries*	206 (26%)	124 (38%)	82 (17%)	21.1%
$\langle Armory\rangle \xrightarrow{debruising\ group} \emptyset$ $\langle Armory\rangle \xrightarrow{secondary\ group} \emptyset$ *Armory with no secondary or debruising groups*	581 (72%)	193 (59%)	388 (80%)	21.1%
$\langle 0\rangle \xrightarrow{has\ count} 1$ $\langle 1\rangle \xrightarrow{includes\ charges} \langle 0\rangle$ $\langle Armory\rangle \xrightarrow{primary\ group} \langle 1\rangle$ $\langle Armory\rangle \xrightarrow{debruising\ group} \emptyset$ $\langle Armory\rangle \xrightarrow{secondary\ group} \emptyset$ *Armory with a primary group of a single charge and no secondary or debruising groups*	340 (42%)	96 (30%)	244 (51%)	21.1%

Pattern	**Total** **N = 807**	**Dering** **N = 325**	**Zurich** **N = 482**	Δf
$\langle 0 \rangle \xrightarrow{has\ count} 1$ $\langle 1 \rangle \xrightarrow{has\ partition\ type} Plain$ $\langle 2 \rangle \xrightarrow{includes\ charges} \langle 0 \rangle$ $\langle Armory \rangle \xrightarrow{has\ field} \langle 1 \rangle$ $\langle Armory \rangle \xrightarrow{primary\ group} \langle 2 \rangle$ $\langle Armory \rangle \xrightarrow{debruising\ group} \emptyset$ $\langle Armory \rangle \xrightarrow{secondary\ group} \emptyset$ *Armory with a plain field and a primary group of a single charge and no secondary or debruising groups*	307 (38%)	83 (26%)	224 (46%)	20.9%
$\langle 0 \rangle \xrightarrow{has\ count} 1$ $\langle 0 \rangle \xrightarrow{tertiary\ group} \emptyset$ $\langle 1 \rangle \xrightarrow{has\ partition\ type} Plain$ $\langle 2 \rangle \xrightarrow{includes\ charges} \langle 0 \rangle$ $\langle Armory \rangle \xrightarrow{has\ field} \langle 1 \rangle$ $\langle Armory \rangle \xrightarrow{primary\ group} \langle 2 \rangle$ $\langle Armory \rangle \xrightarrow{debruising\ group} \emptyset$ $\langle Armory \rangle \xrightarrow{secondary\ group} \emptyset$ *Armory with a plain field and a primary group of single uncharged charge and no secondary or debruising groups*	305 (38%)	83 (26%)	222 (46%)	20.5%
$\langle 0 \rangle \xrightarrow{has\ count} 1$ $\langle 0 \rangle \xrightarrow{tertiary\ group} \emptyset$ $\langle 1 \rangle \xrightarrow{includes\ charges} \langle 0 \rangle$ $\langle Armory \rangle \xrightarrow{primary\ group} \langle 1 \rangle$ $\langle Armory \rangle \xrightarrow{debruising\ group} \emptyset$ $\langle Armory \rangle \xrightarrow{secondary\ group} \emptyset$ *Armory with a primary group of a single uncharged charge and no secondary or debruising groups*	336 (42%)	96 (30%)	240 (50%)	20.3%